

Bone fracture is an injury not uncommon to everyday life. Most of the time it leaves permanent damage and a long period of recovery. If we can understand the mechanics and the process of bone fracture, we can take measures to prevent them. Furthermore, through better understanding of energy transfer in fracture, we can determine the underlying causes of the injuries in order to prevent them from a different perspective. Since human bones are not available for our laboratory study, we use chicken bone in place of human bone in our simulation bone fractures.

This case involves forensic engineering to uncover the circumstances of our situation. This method is used in legal and insurance circles to find the amount of compensation is required and who is liable. Critical analysis of the mathematical model used to describe the situation coupled with our background knowledge will allow us to determine whether or not the provided scenario is the likely cause for injury.

By modeling an impact loading situation on chicken bone using a pendulum and potentiometer, we can study the energy transfer indirectly by measuring voltage and relating it to energy. By modeling a three point bending situation with varying force on chicken bone using an Instron, we can study the mechanical properties of chicken bones and relate our findings to human bones. We sought to determine the amount of energy and/or the force required to fracture the subject's left tibia and to decide if it was possible for him to fracture his tibia in the situation described.

We found the situation given involved a side impact to the man's leg that broke both the fibula and the tibia, as normally happens in impact situations (Sunderland 25). the subject's weight provided the necessary applied force to break the bone at the point of impact. Our results lead us to conclude that the situation given is correct. We speculate that the subject was walking on the porch and collided with a chair on the lateral side of the left leg. The side impact broke the fibula and the force applied was sufficient to break the tibia. Furthermore, startled from the impact, he attempted to readjust his position and subsequently "turned over" his ankle, thus causing the more distal of the two fibular fractures.

INTRODUCTION

Physicians and lawyers routinely attempt to determine the causes of injuries and accidents. A doctor may find it necessary to determine the cause of the injury in order to prescribe the most effective treatment. In a similar manner, a lawyer may place emphasis on uncovering the cause of the accident in order to recreate the scene before a judge or jury. Information about the causes of injuries and accidents allows physicians and lawyers to do their work in a more efficient and focused manner.

Reconstructing the environment of the injury required us to create a model. In this experiment, we used a chicken tibia as a model to study the fracture of human bones. We used an Instron machine, and pendulum setup as a model to study the characteristics of bone bending and fracture of human bones. We can apply the data gathered from these experiments to interpret more complicated situations such as collisions arising from a car accident or sports injuries, or we can simulate situations under three point bending such as a leg or arm being caught in machine that applies constant force such as gear mechanisms that are found in harvesting equipment often associated with farms.

Before testing the specimens for their physical properties, we had to research some background of the bone. Our model represents the bone as a cylinder when in fact an intact bone is not straight at all. The cross-sectional area of the bone varies greatly through the length of the bone along with the thickness. The most important property of the bone is that it is a very heterogeneous tissue consisting of compact and spongy bone, blood cells, fat and marrow (Evans 34). The density of the bone is not equal throughout the bone. The density at the ends of the bone is greater than the center of the bone. All of these conditions make it difficult to test intact bones under pure tension, because the load will not be evenly distributed.

The obese 62 year old man was in a condition to cause injury to himself because of his health condition. His weight puts extra stress on the tibia and fibula, making them more susceptible to failure. The impact occurred when the man hit a chair while walking on his porch. In an attempt to justify the given situation and find the maximum energy transfer involved, we should base our study on the worst case, which is when the man's leg struck the chair perpendicularly (at a 90° angle). The chair was assumed to be propped against a wall so the total energy involved is transferred to the leg. This would allow for the maximum resistive force and concentrate the most energy in one area to fracture the bones.

In our analysis, we sought to determine whether or not the presented scenario is a plausible cause of the injury depicted in the provided x-rays. The forensic analysis in our mathematical model of the situation will allow us to determine whether the case presented is true. This type of experiment gives us a basis to analyze data in a situation where the amount of known facts are limited. The mathematical model approach tests the system in

a disciplined and verifiable manner that can be shown to be true or false. This project will build a foundation to test fractures in other areas of the body under a myriad of conditions. Furthermore, this type of analysis can provide legal evidence for insurance fraud. By studying energy relationships in fracture, we can use this knowledge to predict causes of fracture, and take measures to prevent them. This type of study could result in building safer home and other environments.

METHODS AND MATERIALS

CHICKEN BONE EXPERIMENTS

Due to the limited resources available for experimental testing, the data and results from our Impact Loading and Three Point Bending laboratories were used in combine with those of new experiments. All five chicken legs provided were fractured with flesh at one-third of the length from foot. Three chicken bones were fractured using an impact pendulum/potentiometer setup provided with varying masses. The weighted pendulum was adjusted in order to make impact on the bone at a distance one-third from foot. The pendulum/potentiometer apparatus was calibrated from -90° to $+90^\circ$ (from the vertical) of the pendulum swing to find the change in voltage as degrees changed before the experiment for each bone. Of the three bones that were fractured using the impact pendulum, the first two bones were fractured using 1000g of mass at the end of the pendulum arm, while the third was fractured with 140g of mass. Both the calibration and the experiment were recorded using LabView. The difference in the original and final potential energy of the pendulum arm were calculated to determine the energy required to break each bone. Finally, in order to simulate the re-fracturing of a previous healed bone, the third bone was rejoined using crazy-glue after first fracture followed by a re-fracture under the same conditions. For details on the impact pendulum procedure, please refer to our Impact Pendulum lab report.

The remaining two chicken legs were fractured using the three point bending Instron with different cross-head speeds. Both chicken legs were placed on the Instron in a way so that the point of contact is at a distance one-third from the foot. The first bone was fractured with an Instron cross-head speed of 10 mm/min, while the second was fractured with an Instron cross-head speed of 500 mm/min. The results were recorded using Labview. Again, in order to simulate the re-fracturing of a previous healed bone, the first bone was rejoined using crazy-glue after the first fracture followed by a re-fracture under the same conditions. For details on three point bending procedure using the Instron, please refer to our Three Point Bending lab report.

SITUATION AND MATHEMATICAL MODELING

The situation under study was analyzed using the information and x-ray provided. Mathematical models were then constructed to determine both the energy required to fracture the human tibia and the energy involved in the presented scenario. The amount of energy involved in the collision case provided was calculated using the concepts of kinetic energy with estimated speed of the bone at the point of contact. The mathematical model utilized the concepts of total energy involved in pure bending and energy density, and based on the assumption that the Young's Modulus of human bone is the same as that of

chicken bone. The energy required to break each chicken bone in the impact pendulum experiment was determined using the change in potential energy of the pendulum arm. The energy required to break each chicken bone in the three point bending experiment was determined by calculating the area under each force-displacement curve. The relationship between changes in total energy and the speed of load applied was studied by graphing the total energy required to break each bone with the corresponding speed of load at the contact point (from impact pendulum arm or cross head speed of Instron). Also, the percentage of total applied energy absorbed by the flesh attached to the bone was calculated by comparing the average energy needed to break a chicken bone without flesh and the average energy needed to break a chicken bone with flesh.

Details on situation analysis and mathematical calculations are contained in their respective sections within this report. Finally, based on our mathematical calculations, the energy required to fracture the subject's tibia was compared with the energy involved under the situation provided, and a conclusion of whether or not the given scenario was a plausible cause of injury.

SITUATION ANALYSIS

The situation presented is the complete fracture of the left tibia and fibula. The accident occurred when a 62 year old male weighing 285 lbs. and standing 5ft-10in. bumped into a chair while walking on his porch. The lengths of tibia and fibula were indirectly measured from the x-ray to be 0.365m and 0.38m, respectively. The tibia fractured at 0.140m from the foot, and the fibula fractured in two places: at 0.070m and 0.170m from foot. The average diameter of the tibia from 10 measurements is 0.0272m with a thickness of 0.0063m. And the deformation of the bone is 0.016m. The fracture sections are approximately one-third of the length of the bone from foot. In order to determine whether the man fractured his bone under the situation described above, the case when maximum force, stresses and energy are applied to the bone should be studied. If the maximum applied force and energy transfer given by our model of the situation exceed the threshold of fracture applied force and energy transfer of the tibia, then the subject could very possibly have fractured his tibia in the manner described. However, if the threshold values for the applied force and energy transfer are greater than the ones predicted by our model of the situation, the subject is not likely to have broken his tibia in the presented situation. The tibia is not likely to break under cases where applied force and energy transfer do not exceed the threshold values for fracture.

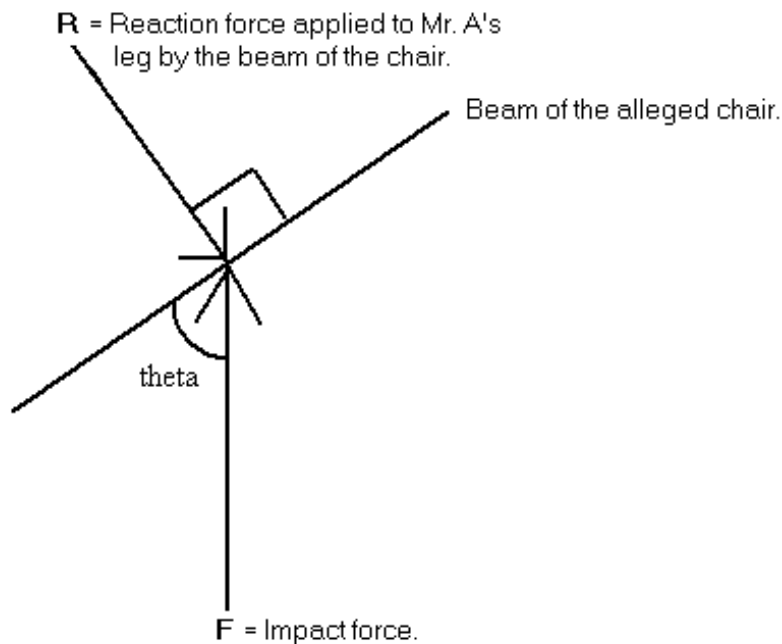
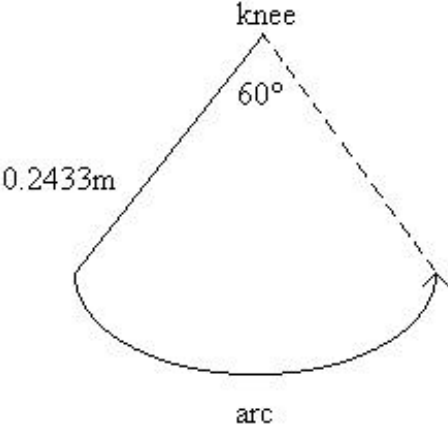


Figure 1. A schematic diagram of the injury scenario illustrating the proposed injury scenario.

The man kicked a chair with his left leg as shown in Figure 1. The angle θ that his leg hit the chair is not known; however, maximum reaction force exerted by the chair on the leg occurs when the angle θ is 90° , which means the man's leg hit the chair straight on. Therefore, in order to study the case where maximum force applied and maximum energy transfer is involved, our calculation will be based on the case when angle θ is 90° . Also, we need to assume that the energy carried by the subject's lower leg before the collision is totally transferred to the bone during the impact. This implies several important limitations. First, there is no energy dissipated as heat. Second, the man's lower leg did not reflect due to impact. Finally, the chair did not deform and remained stationary during and after impact. The amount of energy involved in the impact is calculated based on the kinetic energy of the man's lower leg just before impact. The following series of calculations demonstrates our logic.

	Average Case	Maximum Case
Walking speed was determined from experimental testing on the four members in the group.		
Walking Speed	0.894 m/s	1.565 m/s
We assume that the knee is moving at a speed twice the walking speed when the leg is moving, because each knee moves for half of the total walk time.		
Speed of Knee	1.788 m/s	3.130 m/s
We assume that each step covers the same distance for both cases. This distance is based on the military standard of 30 inches per step.		
Distance of Each Step	0.762 m	0.762 m
Time of each step = $\frac{\text{Distance of each step}}{\text{Walking speed}}$		
Time of Each Step	0.8523 s	0.4869 s
The length of the bone is $0.365m$ and since the point of impact occurred at a distance two-third down from the knee. Thus, the length of the bone from the knee to the point of impact is $\frac{2}{3} \cdot 0.365m$.		
Length of bone from Knee to the Point of Impact	0.2433 m	0.2433 m

<p>We assume that the lower leg swings an angle of 60° every step.</p>  <p>Arc length = $\frac{\text{Circumference}}{6} = \frac{2\pi \cdot 0.2433m}{6} = 0.2548m$</p> <p>The speed of the swing (0.2433m down from knee) is equal to $\frac{\text{Arc length}}{\text{Time of each step}}$.</p>		
<p>Speed of Swing (0.2433m down from knee)</p>	<p>0.2990 m/s</p>	<p>0.5233 m/s</p>
<p>The total speed at the point 0.2433m down from the knee is equal to speed the knee is moving and the speed of swing.</p>		
<p>Total Speed (v)</p>	<p>2.087 m/s</p>	<p>3.653 m/s</p>
<p>We approximate the mass of the lower leg to be 7.5% of total body weight. In this case study, the man's lower leg has a mass of $129.3kg \cdot 0.050 = 6.465kg$.</p> <p>The kinetic energy carried by the man's lower leg is equal to $\frac{1}{2}mv^2$.</p>		
<p>Kinetic Energy of the Lower Leg (K)</p>	<p>14.08 J</p>	<p>46.13 J</p>

CALCULATIONS

ENERGY VS. SPEED RELATIONSHIP

Impact Pendulum

In our impact loading experiment, it is reasonable to assume that the pendulum swings without friction. To calculate the speed of the pendulum just before impact, we apply the law of conservation of energy. The pendulum converts all of its potential energy (U) before the impact (when it is held up and at rest) to kinetic energy (K). Therefore, from conservation of energy, we use:

$$U = K$$

$$mg\Delta h = \frac{1}{2}mv^2$$

$$g\Delta h = \frac{1}{2}v^2. \quad [1]$$

In order to determine the change in height, we need to first calculate the pendulum's center of mass (C_m) using the formula

$$C_m = \sum_0^{\infty} \frac{M_i}{m}$$

where $\sum M_i$ is the sum of moment of each part of the pendulum, and m is the total mass of the pendulum.

The change in height is the length from the top to center of mass. And from equation (1) we can calculate the speed of the pendulum at the center of mass. To determine the speed of the pendulum at the point of impact just before the collision, we use the equation

$$V = R\omega$$

where V is the speed, R is the distance from the center (fixed point), and ω is the angular velocity.

However, we realize that the angular velocity ω is constant at any point along the pendulum arm, so we can calculate the speed at the end of the pendulum arm using the ratios

$$\frac{V_c}{R_c} = \frac{V_e}{R_e} \quad \text{and} \quad V_e = \frac{V_c \times R_e}{R_c}$$

where V_C is the speed at center of mass, V_e is the speed at the end of the arm, R_C is the length from fix point to center of mass, and R_e is the length of the pendulum arm.

Three Point Bending

The cross head speed of the Instron represents the speed of the moving object. Using the above method, we determined the speed of the pendulum just before collision and the cross head speed of the Instron for every bone. Since we find the energy density is independent of the speed of moving object from Figure 2, which shows energy density of bones remains constant as the speed of load increases, we conclude that energy density does not depend on the speed of load. From our previous calculations, the maximum speed of the man's lower leg at the point of collision is faster than both the pendulum arm in our impact loading experiments and the Instron in our three point bending experiments. However, because the energy density is independent of the speed of loading, we use the average energy density (333000 J/m^3) for our mathematical models. The average energy density was calculated from adding the average energy density of chicken bones from impact loading experiments (322000 J/m^3) and the average energy density of chicken bones from three point bending experiments (344000 J/m^3), then the sum was divided by two.

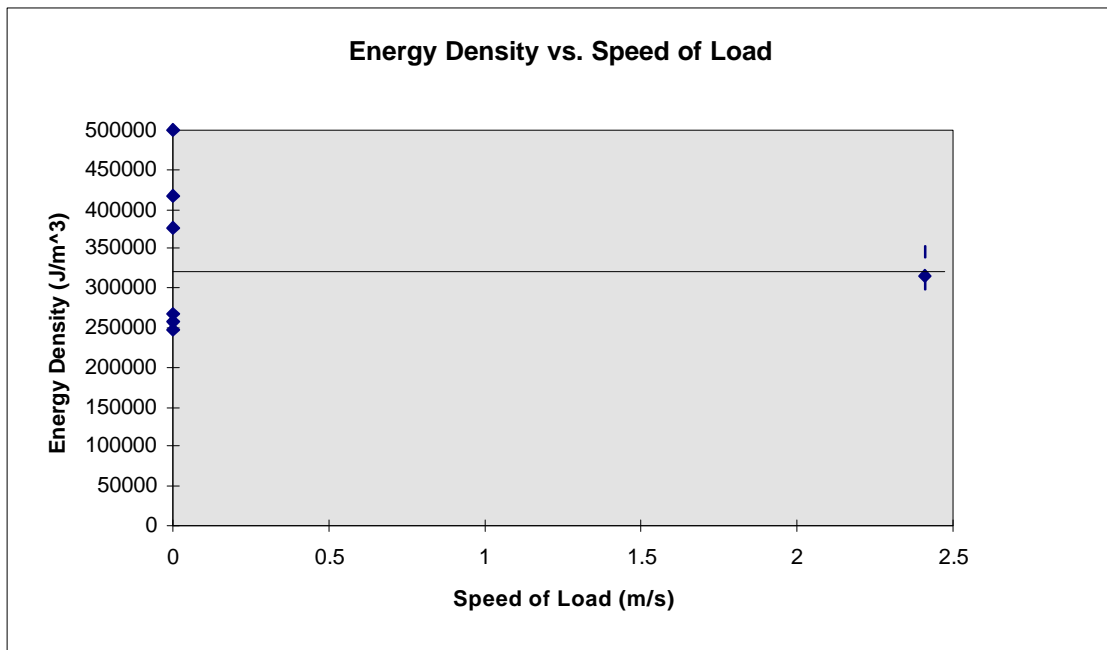


Figure 2. Six chicken bones from three point bending with a cross head speed of 0.000167 m/s and three chicken bones from impact loading with the pendulum arm swings at 2.41 m/s at the point of contact.

PERCENTAGE OF THE TOTAL ENERGY APPLIED TO THE BONE

The average energy required to break a chicken bone without attached muscle is 0.8037J, and the average energy required to break the chicken bone with attached muscle is 2.738J. Again, the average energy required was calculated from adding the average energy required to fracture a chicken bone in impact loading experiments and the average energy required to fracture a chicken bone in three point bending experiments, then the sum was divided by two. We can calculate the percentage of energy the muscle can absorb as follows:

$$\frac{2.738J - 0.8037J}{2.738J} = 0.7065 = 70.65\% .$$

Thus, the percentage of total energy transferred to the bone is: $100\% - 70.65\% = 29.35\%$.

MATHEMATICAL MODELS

Due to the high degrees of limitation and the amount of information provided, we applied two mathematical models to study our case. In the first model, we use the concept of energy density. In the second, we analyze strain energy. For both methods, we need to make the following assumptions:

1. Human tibia is a straight cylindrical shaft with uniform and symmetric cross-section throughout the length of the bone. Each part of the bone is made up of the same material and has the same density; the bone is homogeneous.
2. The bone is subjected to pure bending; shear, torsional, and axial forces do not exist in this scenario.
3. There is no dissipation of energy during the impact.
4. The bone is subjected to uniformly distributed stresses.
5. The muscle attached to the human tibia absorbs the same percentage of total applied energy as that of chicken.
6. We can neglect the inertia of the elements of the bone.
7. The subject's leg did not reflect due to reflection during and after the impact.
8. The contact is on the person's leg. In other words, the person is not wearing long pants. Or, we can assume that cloth does not absorb any impact energy.

STRAIN ENERGY ANALYSIS

In order to study the amount of energy involved in bone fracture, we used the concept of strain energy to determine the energy required to break human tibia. In doing so, we need to add a few more assumptions:

1. The Young's Modulus of human bone is the same as chicken bone.
2. The stress-strain curve obtained from static loading is valid for impact loading as well.
3. The maximum normal stress and shearing stress occur within the proportional limit; the bone is made up of brittle material and is linearly elastic.

The total energy (U) absorbed by the bone, which is also the maximum strain energy (U_s) acquired by the bone, is equal to the loss of kinetic energy ($T = \frac{1}{2}mv^2$) of the moving body. In the pendulum experiment, the loss of kinetic energy is equal to the loss of potential energy of the pendulum. Having determined the maximum strain energy, we may then calculate the maximum load (P_m) that would have produced the same strain energy under three point bending. This will also allow us to determine the maximum stress (σ_m). Strain energy (U_s) is defined as

$$U_s = \int_0^L P dx$$

where P is the load, and L is the length of the bone.

Dividing by the volume ($V=AL$) of the rod, we obtain the strain energy per unit volume or strain energy density (u)

$$\frac{U_s}{V} = u = \int_0^L \frac{P}{AL} dx$$

where A is the cross-sectional area of the bone.

However, V/A is the average normal stress (σ) on the transverse cross section, where V is the magnitude of the shear force and A is the cross-sectional area. Also, dx/L is the average normal strain (ϵ), therefore, we can write

$$u = \int_0^L \mathbf{s} \cdot d\mathbf{e} = \frac{1}{2} \mathbf{s} \mathbf{e}^2 .$$

For values of σ within the proportional limit, we may use Hooke's Law, $\sigma = E\epsilon$, and have

$$u = \frac{Ee^2}{2} = \frac{\mathbf{se}}{2} = \frac{\mathbf{s}^2}{2E}.$$

Therefore, the value of strain energy U of a body can be written as

$$U = \int_0^L \frac{\mathbf{s}^2}{2E} dV.$$

When a bone is bent by a downward force, it experiences stresses in the longitudinal direction or in a direction normal to the cross-section of the bone. The upper portion of the bone is compressed and the lower portion of the bone is in tension. This stress is called flexural stress. In our case study, the human tibia is under pure bending, we have

$$\mathbf{s} = \frac{My}{I}$$

where M is the resistive moment in the cross-section, I is the area moment of inertia about the neutral axis and y is the distance from the neutral axis.

Since we are only concerned about the maximum force and stresses on the bone, we only consider the maximum normal stress, which occur at the upper and lower layers of the bone during pure bending, so y is the outer radius of the bone, and

$$U = \int_0^L \frac{M^2 y^2}{2EI^2} dV.$$

Since $dV = dA dx$,

$$U = \int_0^L \frac{M^2}{2EI^2} \left(\int y^2 dA \right) dx.$$

The integral within the parentheses is equal to the moment of inertia, I , of the cross section about its neutral axis $\left(\int y^2 dA = I \right)$, so we can write

$$U = \int_0^L \frac{M^2}{2EI} dx.$$

The above equation derivation was obtained from Bear and Johnston (1).

From integration, the stiffness value (k) is found to be related to Young's Modulus (E) as follows (refer to Appendix A for these derivations):

$$k = \frac{P}{d} = 55.8 \frac{EI}{L^3} \text{ and } E = \frac{1}{55.8} \left(\frac{kL^3}{I} \right) \text{ and } M = \frac{2}{9} PL.$$

Therefore, we can write the strain energy (U) as

$$U = \int_0^L \frac{2P^2 L^2}{81EI} dx = \frac{2P^2 L^3}{243EI} = \frac{2(55.8)P^2}{243k} = \frac{2(55.8)Pd}{243} = \frac{2(55.8)kd^2}{243}.$$

And the stiffness of human tibia is determined by

$$k_h = \frac{55.8EI}{L^3} = \frac{(55.8)(1.70 \times 10^9) \left[\frac{P}{4} (0.0136^4 - 0.0073^4) \right]}{0.365^3} = 48000 Pa \cdot m.$$

The energy required (U_H) to fracture human tibia is calculated as follows:

$$\frac{U_H}{U_C} = \frac{\left(\frac{2 \cdot 55.8 \cdot k_H d_H^2}{243} \right)}{\left(\frac{2 \cdot 55.8 \cdot k_C d_C^2}{243} \right)} = \frac{k_H d_H^2}{k_C d_C^2}.$$

Thus,

$$U_H = U_C \frac{k_H d_H^2}{k_C d_C^2} = (0.804 J) \frac{(48000 Pa \cdot m) \cdot (0.0160m)^2}{(9.18 \cdot 10^4 Pa \cdot m) \cdot (0.00585m)^2} = 3.15 J.$$

We can then determine the energy required to break the man's tibia under study with muscle attached to it:

$$\frac{3.15 J}{0.2935} = 10.73 J.$$

Therefore, the energy required to break the man's tibia with attached muscle is 10.73J.

STRAIN ENERGY DENSITY ANALYSIS

In a manner similar to our previous analysis, we used the concept of strain energy density to determine the energy required to break human tibia. In doing so, we need to add a few more assumptions:

1. Strain-energy density is the same for chicken bone and human bone.
2. The bone is under pure bending; shear, torsional and axial forces do not exist.
3. The stress-strain curve obtained from static loading is valid for impact loading.

From the strain energy analysis, we had

$$u = \int_0^L \mathbf{s} \cdot d\mathbf{e} = \frac{\mathbf{s}\mathbf{e}^2}{2}$$

The value of strain energy density (u) is obtained by integrating from 0 to ϵ_{\max} , where ϵ_{\max} is the normal strain at fracture. The value (u) represents the energy per unit volume required to fracture the bone. From Figure 3., we can see that the total energy increases almost linearly as volume increases.

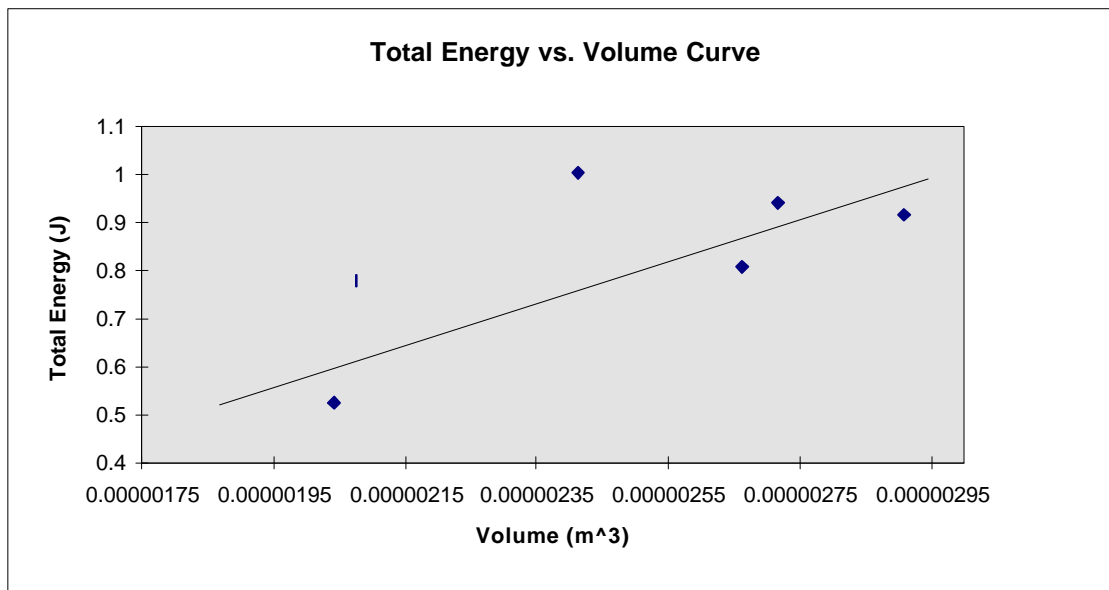


Figure 3. Total energy required to fracture a chicken bone increases as the volume of the chicken bone increases. The slope of the best fit line is approximately 375000 J/m^3 , which represents the energy density of chicken bone.

Therefore, it is valid to use energy density and the volume of the bone to calculate the total energy required to fracture the bone. Assuming the energy per unit volume required to fracture human bone (energy density) is the same as the energy per unit volume required to fracture chicken bone, we can calculate the energy required to break a human

long bone by knowing the strain-energy density (u) of chicken bones and the volume of both the human bone and the chicken bone. The average energy density of chicken bone is

$$\frac{344000 \text{ J/m}^3 + 322000 \text{ J/m}^3}{2} = 3330000 \text{ J/m}^3 .$$

In Figure 3, the slope of the best fit curve is approximately 375000 J/m^3 , which represents energy density, agrees closely with our calculated average energy density (333000 J/m^3) for chicken bone. In our model, the human tibia has a volume of

$$\text{TibiaVolume} = 0.365m \times p[(0.0136m)^2 - (0.0073m)^2] = 1.511 \times 10^{-4} m^3 .$$

Assuming the strain-energy density of human long bone is the same as that of chicken, the energy required to break the human tibia without muscle can be calculated as follows:

$$(\text{Volume of the bone}) \cdot (\text{Strain-energy density}) = \text{Energy required},$$

thus the energy required to break the bone is 126.7J. We further assume that the percentage energy absorbed by muscle around human tibia is the same as that of chicken. From that assumption, we can determine the amount of energy required to break the man's tibia under study with attached muscle:

$$\frac{50.35J}{0.2935} = 171.6J .$$

Therefore, the energy required to break the man's tibia with attached muscle is 171.6J.

SUMMARY TABLE

	<i>MODEL 1: Strain Energy and Stiffness Relation</i>	<i>MODEL 2: Energy Density</i>
<i>Key Assumption</i>	Young's Modulus of human bone is the same as the Young's Modulus of chicken bone.	The energy required per unit volume (energy density) to fracture the human bone is the same as that of chicken bone.
<i>Calculated Energy Required to Fracture the Subject's Tibia</i>	10.73 J	117.6 J
<i>Is the Situation Provided Valid?</i>	Yes	No

RESULTS

The subject's tibia at a distance one-third of the length from foot is estimated to move at a speed of 2.087 m/s during normal walking and 3.653 m/s during rapid walking. With the assumption that the lower leg has a mass of approximately 5% of the total body mass, the maximum kinetic energy carried by the lower leg at the point of contact was found to be 14.08 J for normal walking and 46.13 J for rapid walking. The maximum speed of the man's lower leg at the point of collision was faster than both the pendulum arm (2.41 m/s) in our impact loading experiments and the Instron (0.000167 m/s) in our three point bending experiments. From our experiments using chicken bone, energy density was found to be independent of the speed of loading (Figure 2). In addition, we calculated that the percentage of total energy applied to the bone is 29.35%, which means the flesh attached to the bone absorbs 70.65% of total applied energy. We determined the average Young's Modulus of chicken bone to be 1.70 GPa.

Two mathematical models were studied. In the first model, we found that the energy required to fracture a bone increases linearly as volume increases and the average energy density is 333000 J/m^3 . Assuming that the energy density of human tibia and that of chicken bone are the same, we found the energy required to fracture the subject's tibia to be 171.6 J. In this case, the energy required to fracture the subject's tibia (171.6 J) is much higher than the maximum energy involved under the situation provided (46.13 J), and the scenario given could not be true. In the second model, which we assumed the Young's Modulus of human bone and that of chicken bone are the same, the stiffness value of human bone was found to be 48063.4 Pa-m, which is about one-half the stiffness value of chicken bone (91800 Pa-m). Using the calculated stiffness values and the relationship between total energy and stiffness, the energy required to fracture the subject's tibia was found to be 10.73 J. In this scenario, the amount of energy required to fracture the subject's tibia (10.73 J) is much lower than the maximum energy involved under the situation provided (46.13 J), thus the scenario given may very well be true. However, knowing that the human bone could not fracture with less absorbed energy than chicken bone, we concluded that we can not assume the Young's Modulus of human bone and the Young's Modulus of chicken bone to be the same. We subsequently concluded that the second mathematical model is invalid. Thus, the situation presented is unlikely to be the cause of injury based on our first mathematical model.

DISCUSSION

From our situation analysis, the collision resembled impact loading more than three point bending. This can be easily seen by calculating the speed of loading in impact loading, three point bending and in the incident given (refer to the situation analysis and calculation). Another way to understand that the incident is quick impact loading rather than slow bending is by examining how the bone fractured. Figure 4 shows an oblique tibial fracture (the tibia fractured at an angle from the transverse cross-section), which resembles bones fractured from impact loading (Figure 5). Figure 6 shows a bone fractured under slow bending. Every bone from our three point bending experiment fractured at a cross-section along the transverse cross-section (perpendicular to the longitudinal cross-section) of the bone.

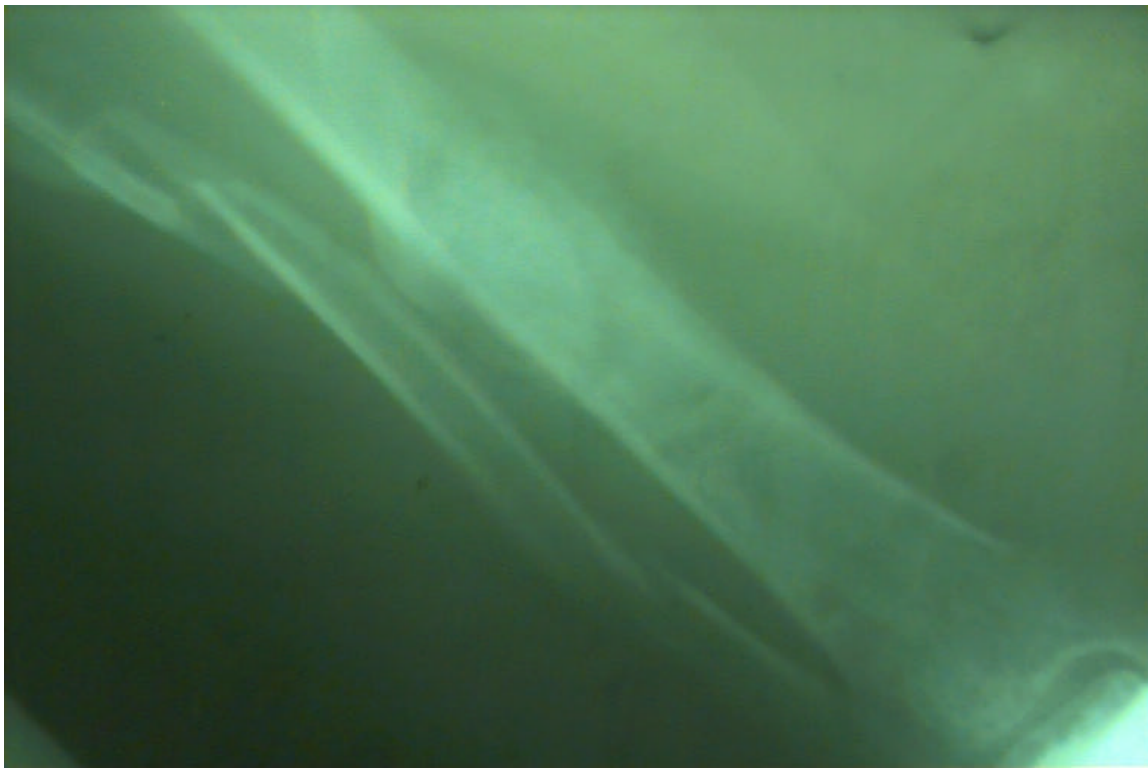


Figure 4. Enlarged X-ray of the subject's fractured left tibia and fibula. The tibia fractured at an angle from the transverse cross-section.



Figure 5. Enlarged picture of a chicken bone fractured using impact pendulum. The bone fractured at an angle from the transverse cross-section.



Figure 6. Enlarged picture of a chicken bone fractured using three point bending Instron. The bone fractured along the transverse cross-section (perpendicular to the longitudinal cross-section).

Our first mathematical model is based on multiplying energy density with volume to determine the energy required to break a similar bone of any volume. Using the results from our Impact Loading lab, we were able to calculate the average energy density required to break a chicken bone without flesh under impact collision. We first determined the total energy needed to break each bone using the difference in the pendulum arm's potential energy before and after impact, then divided it by the volume of each bone (only the bone shaft) to find the energy density. Similarly, using the results

from our Three Point Bending lab, we were able to calculate the average energy density required to break a chicken bone without flesh under slow loading condition. However, there are two ways to determine the energy density of each bone fractured from Instron. The first method estimates the area under the force-displacement curve, which represents the total energy applied to the bone, then divides it by the volume of each bone to determine the energy density. The second method is used to estimate the area under the stress-strain curve, which itself represents the energy density of the bone. Applying both methods, we found that the average energy density calculated by the two methods are inconsistent with one another; the energy density obtained from calculating the area under the stress-strain curve is higher than that determined from the area under the force-displacement curve. This difference is contributed by assuming the bone is a hollow cylinder, where it actually contains haemopoietic tissue of the marrow (12, p. 108). In order to combine the results with those obtained in impact loading experiments, we calculate energy density using force-displacement curves, which is the same method as we utilized in impact loading experiments (total energy/volume). From Figure 2, we can see that the energy density is independent of the speed of loading. Therefore, we average the mean energy density from impact loading and the mean energy density from three point bending to find the average energy density under all conditions.

Multiplying the average energy density with the volume of the subject's tibia shaft, we estimated the energy required to break the tibia to be 171.6 J. Comparing to the estimated maximum energy involved when the man hit a chair while walking (46.13 J), the energy required is almost 4 times the maximum energy available in the incident. Therefore, we concluded the scenario provided is unlikely to be true.

In our second mathematical model, we tried to relate the stiffness value to the total energy required to fracture a bone by assuming the Young's Modulus of human tibia is the same as the Young's Modulus of chicken bone. From our calculations, we found the stiffness of human bone to be about half the stiffness of chicken bone, and subsequently we found the energy required to fracture is 10.73 J. In this case, the energy required is about one-fourth of the maximum energy available in the incident. However, having a lower stiffness value for human bone than that of chicken bone, we are implying that human bone is easier to fracture than chicken bone. This cannot be true. Therefore, we concluded that we cannot assume the Young's Modulus of human bone is the same as the Young's Modulus of chicken bone, and this model is invalid.

The two bones that were rejoined together after first fracture were fracture again under the same condition in the first fracture. For the bone fractured with impact pendulum, the second fracture required only one-ninth of the average energy required in first fracture. For the bone fractured with instron, the second fracture required about one-third of the average energy required in first fracture. A bone properly healed from a fracture will most likely to break at a different cross-section on a second fracture, because the properly healed section is usually stronger than other parts of the bone (13, p. 637), because the scar tissue is stronger than the original bone tissue. Both rejoined bones fractured at exactly the same cross-section as their respective first fracture, so they were

not properly healed. Not enough time was allowed to perform enough trials to obtain a proper correlation. However, we can certainly conclude that a bone not properly healed from the first fracture would require much less energy to fracture again at the same position.

Due to the limited number of chicken bone allowed, we were unable to perform enough of any one experiment for our data to be “statistically” relevant. Our analysis was limited by the number of chicken bones at our disposal. Also, we assumed that the bone was homogeneous throughout our models and calculations, when, in fact, it is a “very heterogeneous structure consisting of compact bone, spongy bone, blood, marrow and fat with different density at the end of the bone and at the middle of the bone (4, p. 113). Moreover, in our model, we assumed the bone is under pure bending; however “because bones are not straight, bending is apt to be accompanied by some torsion and possible axial stress. If torsion occurs, tensile, compressive, and torsional shearing stresses are also acting on the bone during a test” (Evans 35). In addition, because Young’s Modulus depends on factors such as weight bearing, density and thickness of the bone. Also, for a bone, Young’s Modulus and ultimate strength actually increases as the speed of loading increases. (4, p. 112) Therefore, we are allowing much uncertainty by assuming the Young’s Modulus of human bone is the same as that of chicken bone, along with the assumption that Young’s Modulus is independent of the loading speed. If the subject had shown signs of osteoporosis, it would dramatically affect the stiffness and Young’s Modulus values of the bone, but for our study, we assume osteoporosis does not occur. For an explanation of the effect of osteoporosis, please refer to Appendix B.

This experiment helped us to form a basis to analyze data in a situation where the amount of known facts are limited. The mathematical model approach tested our system in a disciplined verifiable manner that can easily be shown to be true or false. This project lays the groundwork to test fractures in other areas of the body under a variety of conditions. Forensics can predict the actual cause of an injury. Our study can be used to relate energy transfer in fracture, and aid in prevention of injuries. We can also use this approach to help us make a safer living environment.

LIMITATIONS OF OUR ANALYSIS

In this analysis, our models were limited by the heterogeneous makeup of the bone tissue. We modeled the bone as a body homogeneous when, in fact, it is a “very heterogeneous structure consisting of compact and spongy bone, blood, marrow, fat, etc. Therefore it is difficult to test intact bones under pure tension or compression since the load will not be uniformly distributed over a constant cross-sectional area” (Evans 34). The behaviors of heterogeneous structures are almost impossible to predict accurately in stress-strain analysis because of their lack of uniform composition. Accurate analysis of heterogeneous materials is beyond the scope of our current knowledge. Recognizing this fact, we modeled the bone as a homogeneous body, thus, we were able to use previously derived formulas dealing with forces evenly distributed through cross-sectional areas.

Our models also approximate the bone to be a hollow cylinder. From tactile analysis of the chicken bones and examination of various biology and anatomy texts, we know that long bones are not cylindrical in shape. Yet, the cross-sectional area of most bones do not adhere to the layout of any regular geometric shape. For our purposes, circular cross-sections allowed for reasonable approximations for values such as stress and strain endured by the bone.

Additionally, cylinders have two radii. We were able to accurately measure various widths of the subject’s bone directly from the x-ray, thus providing a reasonable estimate of the outer radius for our model. However, the inner radius and the thickness of the subject’s bone were more difficult to approximate. After careful inspection, we were able to approximate the thickness of the bone in the same manner as we had found the outer radius. Again, we approximated the bone to be a uniform cylinder. From the fractured chicken bones, we know that the thickness of the wall of the bone is not uniform. On the other hand, the thickness cannot necessarily be simplified into some mathematical function. Thus, we approximated the thickness of the bones to be uniform.

Another problem that we encountered was the macroscopic nature of the specimens that we tested. All of our specimens were taken straight from the meat. Thus, the specimens were irregularly shaped. Many of the studies that were found in our research “manufactured” their own specimens by cutting them from the bulk material. For instance, “in order to investigate the relation between mode of failure and impact particle velocity Bird et al. (1968) made impact tests on specimens of cortical bone from fresh beef femurs” (Evans 151). Instead of testing the bone as a whole, they cut test “jigs” from the beef bone of known dimensions of radius and length. This avoids the problem of approximating shapes and thickness, etc.

Furthermore, our specimens had little connection to one another. The variations from bone to bone between testing times, such as amount of water lost and the temperature at which they were tested added, introduced more variables than we could

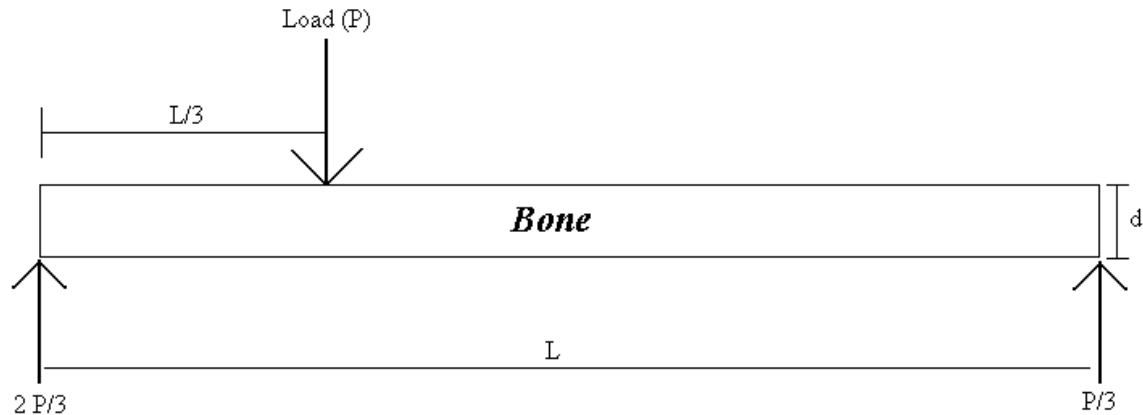
accurately control. The effects of these variations are negligible when taken into account with the other limitations of our analysis.

Finally, we were limited in resources, mainly the number of chicken bones at our disposal. We were unable to perform enough of any one experiment for our data to be “statistically” relevant. Our data points were scattered and we had to further approximate straight lines given limited data. We were unable to perform enough tests to allow our data to “cluster” around any particular values.

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APPENDIX A: DERIVATION OF FLUXURAL STRESS AND STIFFNESS



STRESS

From equilibrium, we can easily calculate the resistive moment (M) at the cross section where the load applies. The moment M can be calculated using the right side of the beam or the left side of the beam, the answer will be the same.

$$M = \frac{P}{3} \times \frac{2L}{3} = \frac{2P}{3} \times \frac{L}{3} = \frac{2PL}{9}$$

where P is the applied load and L is the length of the portion of the beam supported between the two base supports. For a beam under pure bending as shown in the diagram1, the flexural stress (σ) is defined as:

$$s = \pm \frac{My}{I}$$

where y is the distance between the neutral axis and the point at which the stress is in concern, and I is the area moment of inertia. For a hollow cylinder, I can be found (2) as:

$$I = \frac{P}{4}(r_o^4 - r_i^4)$$

where r_o is the outer radius and r_i is the inner radius.

DISPLACEMENT

Maximum deflection formula can be obtained from the table of deflection in most mechanic text. The formula below (3) is applicable for our case:

$$d_{\max} = \frac{Pb(L^2 - b^2)^{3/2}}{9\sqrt{3}EI} \text{ occurs at } x = \sqrt{\frac{L^2 - b^2}{3}}$$

where $b = L/3$ in our case, so we have:

$$d_{\max} = \frac{8^{3/2} PL^3}{729\sqrt{3}EI} = 0.01792 \frac{PL^3}{EI} . \quad [1]$$

STIFFNESS

Combining Hooke's Law with equation [1], stiffness(k) can be related to Young's Modulus (E) by the equations

$$k = \frac{P}{d} = \frac{55.8EI}{L^3} \text{ OR } E = \frac{kL^3}{55.8I}$$

APPENDIX B: EFFECTS OF OSTEOPOROSIS

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